

Performance Analysis of Low-Complexity Welch Power Spectral Density for Automatic Frequency Analyser

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ABSTRACT

The aim of this paper is to investigate the performance of the Low Complexity Welch Power Spectral Density Computation (PSDC). This algorithm is an improvement from Welch PSDC method to reduce the computational complexity of the method. The effect of the sampling rate and the input frequency toward to accuracy of frequency detection is being evaluated. From the experiment results, sampling rate nearest to the twice of the input frequency provides the highest accuracy which achieved 99%. The ability of the algorithm to perform complex signal also has been investigated.

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1. INTRODUCTION

Frequency based sensor have grown rapidly in popularity. Hence, the requirement of frequency analysis also highly increases. In the frequency analysis, the common process is the spectral density calculation. There are many methods of spectral density calculation been proposed, namely as Robust Spectral Density Estimation (RSDE), Independent Component Analysis (ICA), Multitaper Power Spectral Density Estimation (MPSDE), Low-complexity Welch Power Spectral Density (LCWPSD), B-Spline Windows Power Spectral Density Estimation and etc. [1]–[13]. Among the method listed, LCWPSD is the most suitable for biosensor processing. Hence, it has been proposed to apply in biosensor application. Optimum parameter are required to enhance the performance of LCWPSD. Next sections are going to discuss the details for LCWPSD.

2. LITERATURE REVIEW

In this section, the propose method will be discussed in details on how the spectral density of the signal is detected. LCWPSDC [4] is a modified method from the Welch PSDC [14] in term to reduce the complexity of computational. Figure 1 illustrated the flow of the LCWPSDC. Input signal first will be segmented into them with the length of N . Then, the segment with the length of N will be divided into two windows which length of $N/2$. After dividing into two windows, the FFT will be applied to each window, the product called $N/2$ -FFT. The $N/2$ -FFT from the window at the same segment will be merge again become N -FFT. The mathematical derivation of FFTs process is present as below.

The signal of every segment of the input signal as $x[n]$, where the $n=0, \dots, N-1$ of length of N . Given the FFT of $x[n]$ as Eq. 1.

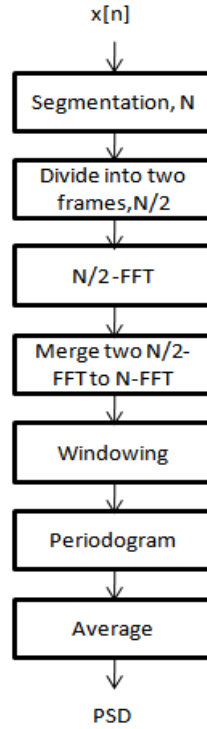


Figure 1. Flow of the Low Complexity Welch PSDC

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} nk} \quad (1)$$

Eq. 1 was substituted with $k=s+2u$ and $n=l+mM$, where $M=N/2$, $s=\{0,1\}$, $u=0, \dots, M-1$, $m=\{0,1\}$, and $l=0,1, \dots, M-1$, we get ,

$$X_1(k) = \sum_{n=0}^{M-1} x[n] e^{-j \frac{2\pi}{M} nk} \quad (2)$$

$$X_2(k) = \sum_{n=0}^{M-1} x[n+M] e^{-j \frac{2\pi}{M} nk} \quad (3)$$

$$X_3(k) = \sum_{n=0}^{M-1} x[n] e^{-j \frac{\pi n}{M}} e^{-j \frac{2\pi}{M} nk} \quad (4)$$

$$X_4(k) = \sum_{n=0}^{M-1} x[n+M] e^{-j \frac{\pi n}{M}} e^{-j \frac{2\pi}{M} nk} \quad (5)$$

To merge two N/2-FFT into N-FFT Eq. 6 and Eq. 7 are applied.

$$X(2u) = X_1(k) + X_2(k) \quad (6)$$

$$X(2u+1) = X_3(k) - X_4(k) \quad (7)$$

After that the N-FFT will be applied with windowing process. In windowing process, Fractional-delay (FD) finite impulse response (FIR) filter was applied. The coefficients of the FD FIR are determined using Eq. 8, where D is the delay with non-integer. In this case, delay is set as 0.5.

$$h[n] = \begin{cases} \text{sinc}\left(n-D-\frac{N}{2}+1\right), & 0 \leq n \leq N-1 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

After windowing process the periodogram of each segment is determined. The last stage is to calculate the average of the periodogram from all the segments as the PSD of the signal. The LCWPSDC algorithm will simulate thru MATLAB.

3. METHODOLOGY

In Section 2, the LCWPSDC method was discussed. In this section, the experiment will evaluate the effect of the sampling rate and input frequency as well as the ability of algorithm to process complex signal. The experiment will be done using MATLAB simulation. Hence, three experiments will be conducted. The sinusoidal signals are selected as the input signal to perform the experiment. Figure 2 illustrated the original input signal of 20Hz frequency.

The first experiment is to determine the relationship between the sampling rate and the accuracy of the algorithm. This experiment used the 20Hz of sinusoidal signal as the input signal. The sampling rate are varied from 50Hz until 250Hz with each increment of 50Hz. Figure 3 illustrated the sampled input signal of 20Hz frequency using 150Hz sampling rate.

The second experiment is to determine the relationship between the input frequency and the accuracy of the algorithm. This experiment is set the sampling rate fix at 150Hz while the input sinusoidal signal are varied from 10Hz until 50Hz with each increment 10Hz.

The third experiment is to study the ability of the algorithm to perform frequency detection from complex signal. This experiment is performed with complex input signal which is the summation of 10Hz, 30Hz and 50Hz sinusoidal signal. Figure 4 illustrated the sampled complex input signal of 10Hz, 30Hz and 50Hz using 150Hz sampling rate. Table 1 and Table 2 summarize the experimental setup.

Table 1. Experimental setup of first experiment

Input frequency	Sampling Rate
20	50
20	100
20	150
20	200
20	250

Table 2. Experimental setup of second and third experiment

Input frequency	Sampling Rate
10	150
20	150
30	150
40	150
50	150
Complex signal	150

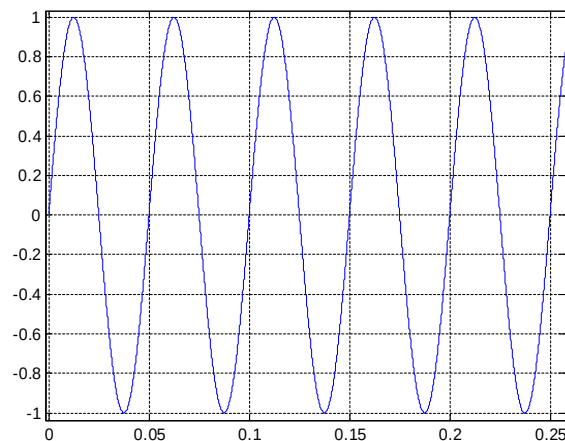


Figure 2. Original input signal of 20Hz frequency

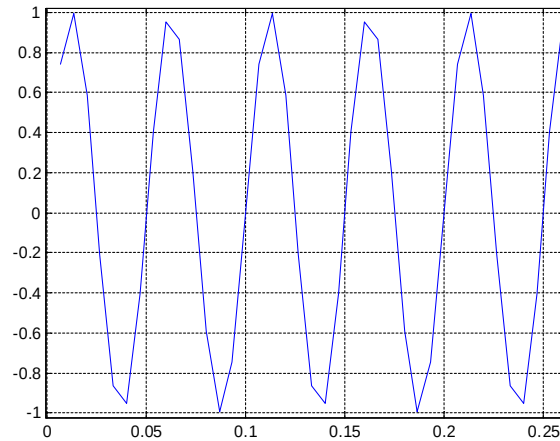


Figure 3. Sampled input signal of 20Hz frequency using 150Hz sampling rate

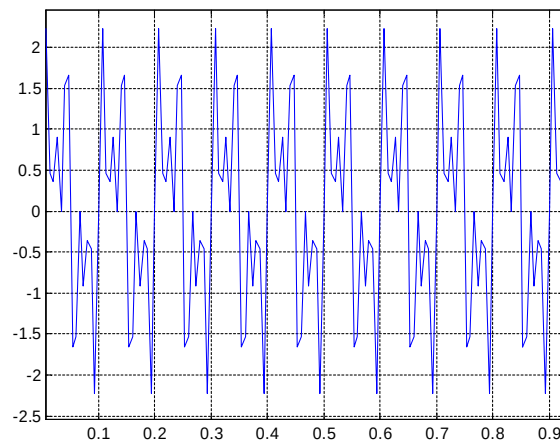


Figure 4. Sampled complex input signal of 10Hz, 30Hz and 50Hz using 150Hz sampling rate

4. RESULTS AND DISCUSSIONS

As the experiment discussed in Section 3, the results will be discussed in this section. All three experiments results will be discussed in detail.

Figure 5 illustrated the spectral density detection that using algorithm LCWPSDC with 150Hz sampling rate for input sinusoidal signal of 30Hz frequency.

Table 3 shows the results of the first experiment to study the effect of sampling rate to the accuracy of the algorithm. From the results, sampling rate at 50Hz is able to achieve 99.17% accuracy of frequency detection whereas sampling rate at 250Hz obtained 96.24% accuracy of frequency detection. The results show that the accuracy are decreases as the sampling rate increases. This trend is observed because sampling rate will used as the scale during FFT and larger scale to measure the value will decrease the accuracy. Hence, the lowest sampling rate provides the highest accuracy and the accuracy is decreasing with the increasing of sampling rate.

Table 4 shows results for the second experiment to study the effect of input frequency to the accuracy of the algorithm. From the results, lowest input frequency with 10Hz has 94.53% accuracy of frequency detection. The highest input frequency with 50Hz has 98.93% accuracy of frequency detection. The results show that in accuracy is increasing while the input frequency is increasing. Base on the Nyquist rate, the lowest sampling rate must be at least twice of input frequency to avoid aliasing[15]. Hence, concluded from the first and second experiment, the sampling rate nearest to twice of input frequency will provide the highest accuracy.

Figure 6 illustrated the results of the third experiment to study the ability of algorithm on complex signal. From the results, it showed that the algorithm was able to detect the complex signals that consist of summation of 10Hz, 30Hz and 50Hz with the frequencies of 10.55Hz, 30.62Hz and 50.54Hz. This results are compared to Table 4 where when each signal is detected individually for 10Hz, 30Hz and 50Hz input

frequencies are 10.55Hz, 30.62Hz and 50.54Hz respectively. This shows that the algorithm detect the complex signal are same as the signal has been detected individually. Hence, it proves that the algorithm are able to perform on complex signal.

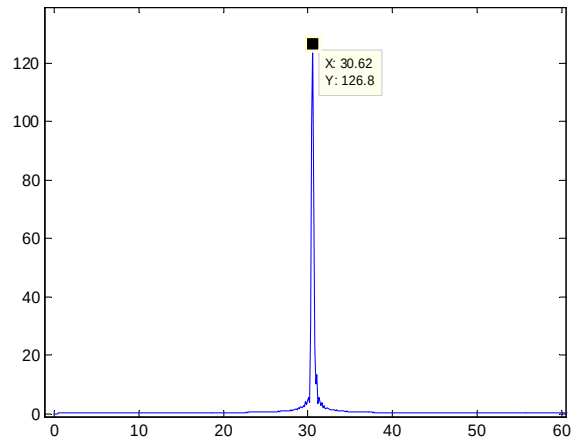


Figure 5. The results of 30Hz sinusoidal signal as the input signal using 150Hz sampling rate

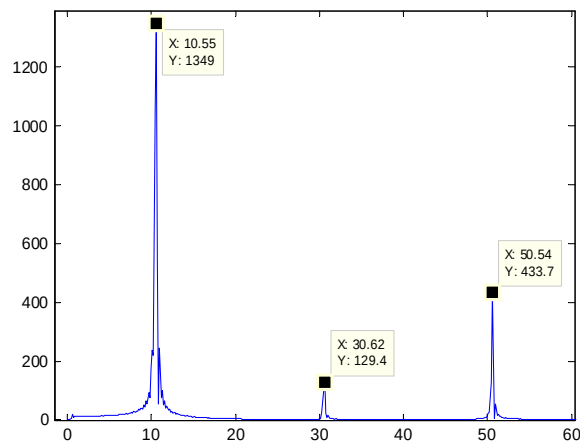


Figure 6. The results of summation of 10Hz, 30Hz and 50Hz sinusoidal signal as the complex input signal using 150Hz sampling rate

Table 3. Experimental The frequency detection on fix 20Hz input frequency

Sampling Rate	Detected frequency	Accuracy (%)
50	20.16602	99.71
100	20.31250	98.44
150	20.36133	98.19
200	20.70313	96.48
250	20.75195	96.24

Table 4. The frequency detection on fix 150Hz sampling rate

Sinput Frequency	Detected frequency	Accuracy (%)
10	10.546875	94.53
20	20.361328	98.19
30	30.615234	97.95
40	40.576172	98.56
50	50.537109	98.93

5. CONCLUSION

Title of manuscript is short and clear, implies research results (First Author)

In this paper, the LCPSDC has been reviewed. The effect of the sampling rate of the algorithm to the accuracy of algorithm has been investigated. The relationship of the sampling rate and the accuracy are inversely proportional. The effect of the input frequency to the accuracy of the algorithm also has been investigated. The relationship of the input frequency and the accuracy is directly proportional where the accuracy is higher with the increasing of input frequency. The experiment has determined that LCPSDC has the ability to perform complex signal too. Further experiment will be done to the LCPSDC in future to investigate the optimum parameter for apply in biosensor signal application.

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